The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: September 30, 2021 Course: EE 313 Evans

Name: ____
Last, First

- This in-person exam is scheduled to last 75 minutes.
- Open books, open notes, and open class materials, including homework assignments and solution sets and previous midterm exams and solutions.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please mute all computer systems.
- Please turn off all phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- <u>**Fully justify your answers**</u>**.** If you decide to quote text from a source, please give the quote, page number and source citation**.**

Problem	*Point Value*	*Your score*	*Topic*
1	22		Sampling Sinusoids
2	24		Squaring System
3	30		AM Radio
4	24		Fourier Series Properties
Total	100		

Problem 1.1 *Sampling Sinusoids*. 22 points.

Consider the sinusoidal signal $x(t) = \sin(2 \pi f_0 t + \theta)$ for continuous-time frequency f_0 in Hz.

We then sample $x(t)$ at a sampling rate f_s in Hz to produce a discrete-time signal $x[n]$.

(a) Derive the formula for $x[n]$ by sampling $x(t)$. *6 points.*

(b) Based on your answer in part (a), give a formula for the discrete-time frequency $\hat{\omega}_0$ of $x[n]$ in terms of the continuous-time frequency f_0 and sampling rate f_s. Units of $\hat{\omega}_0$ are in rad/sample. *6 points*.

(c) For continuous-time frequency f_0 = 392 Hz and sampling rate f_s = 48000 Hz,

i. What is the smallest discrete-time period in samples for $x[n]$? Why? *5 points.*

ii. How many continuous-time periods of $x(t)$ are in the smallest discrete-time period of $x[n]$? Why? *5 points.*

Problem 1.2 *Squaring System.* 24 points.

Consider the signal $x(t) = \cos(2\,\pi\, f_1\, t) + \cos(2\,\pi\, f_2\, t)$ where $f_1 \neq f_2$.

The signal $x(t)$ is input to squaring system to produce the output $y(t) = x^2(t)$.

(a) Write $y(t)$ as a sum of cosines. What non-negative frequencies are present? Leave your answers in terms of f_1 and f_2. *9 points.*

(a) For $f_1 = 110$ Hz and $f_2 = 220$ Hz, write the signal $y(t)$ using the Fourier series synthesis formula

$$y(t) = \sum_{k=-N}^{N} a_k e^{j2\pi(k f_0)t}$$

i. What is the largest possible positive value of f_0? *3 points.*

ii. What is the value of N? *3 points.*

iii. Give the values of all the Fourier series coefficients a_k for $k = -N, \ldots, 0, \ldots, N$. *9 points.*

Problem 1.3. *AM Radio.* 30 points**.**

Each AM radio station operates at a fixed broadcast frequency f_c in the range 530 kHz to 1700 kHz.

Amplitude Modulation. An AM radio transmitter converts an audio signal $x(t)$ to a radio signal $s(t)$ via

$$s(t) = (\, x(t) + A \,) \cos(2\,\pi\, f_c\, t)$$

where A is a constant chosen to be greater than the maximum value of $|\, x(t) \,|$.

For this problem, use $f_c = 1300$ kHz and $x(t) = \cos\left(2\,\pi\, f_1\, t + \frac{\pi}{4}\right)$ where $f_1 = 700$ Hz.

(a) Draw the spectrum for $x(t)$. *6 points.*

(b) Draw the spectrum for $s(t)$. *6 points.*

Amplitude Demodulation. We will use undersampling in the demodulation process to convert the received AM radio signal $s(t)$ to an audio signal $\hat{x}(t)$.

(c) Sample the AM radio signal $s(t)$ at a sampling rate of $f_s = 650$ kHz to create the signal $s[n]$. Give a formula for $s[n]$. *6 points.*

(d) Draw the spectrum for $s[n]$ for discrete-time frequencies $-5\pi < \hat{\omega} \leq 5\pi$. *6 points.*

(e) What continuous-time frequencies would be present when reconstructing a continuous-time signal from $s[n]$? Please include negative, zero, and positive frequencies, if present. *6 points.*

Problem 1.4. *Fourier Series Properties.* 24 points.

The continuous-time Fourier series has several properties.

For example, if $y(t) = A\,x(t)$ and $x(t)$ is periodic with fundamental period f_0 and Fourier series coefficients a_k, then the Fourier series coefficients b_k for $y(t)$ can be found using $b_k = A\,a_k$:

$$y(t) = A\,x(t) = A \sum_{k=-\infty}^{\infty} a_k\, e^{j2\pi(kf_0)t} = \sum_{k=-\infty}^{\infty} A\,a_k\, e^{j2\pi(kf_0)t}$$

For the following expressions, derive the relationship between the Fourier series coefficients b_k for $y(t)$ and the Fourier series coefficients a_k for $x(t)$ where

$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$$

(a) $y(t) = x(t - T)$. *6 points.*

(b) $y(t) = x(-t)$. *9 points.*

(c) $y(t) = \cos(2\,\pi\, f_0\, t)\, x(t)$. *9 points.*